Utility-based Models and Decision Making Problems for Selected Network Processes

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Abstract

Purpose – The concept of utility was the first time applied in Economics. The paper reports its usefulness for the decision making in complex technological systems, in general and in computer networks, in particular. Three selected decision making problems are considered, corresponding solution algorithms are explained and results of numerical experiments are presented for the selected real-world case study.

Design/methodology/approach – Referring to similar decision making problems in Economics, three problems of different time horizon are investigated: strategic investment planning, short-term network rate allocation and on-line network operating. Deterministic and uncertain versions are taken into account, and the latter one is handled more thoroughly. The formalism of uncertain variables is used to represent the parameter uncertainty which concerns users' demands for service in computer networks as well as network links' capacities. Corresponding optimization tasks are presented. Numerical experiments concerning a part of the computer network Pionier working in Poland confirmed the usefulness of the solution algorithms proposed.

Findings – The carried out numerical experiments verified the importance and worth of the decision making algorithms for the Pionier computer network. It particularly concerns the game theory based algorithm solving the on-line network operating problem which enables calculating the rates for computer links distinctly, i.e. separately for every link.

Research limitations/implications – More case studies should be considered to formulate more general corollaries. The application of utility concept for wireless sensor networks needs further studies on solution algorithms..

Practical implications – The results can be directly applied to a class of modern computer networks, e.g. content delivery networks, self-management networks, context aware networks, multilevel virtual networks.

Originality/value – The paper presents the unified and systematic approach for individual results previously obtained, and it considers one case study.

Keywords Utility, Management, Decision making, Computer networks, Uncertain systems, Optimization

Paper type Research paper

1. Introduction

A concept of utility introduced by Daniel Bernoulli in XVIII century has been used in the field of Economics for a long time (see e.g. von Neumann and Morgenstern, 1953, Nicholson and Snyder, 2008). The utility expresses a satisfaction experienced by a consumer of a good or service. Despite a consumer's utility is hard to measure directly, it was recognized that the utility can be represented by an increasing, strictly concave and continuously differentiable function. It made possible to quantify in a numerical way such a soft concept as preferences over some services or goods. It is practically impossible to mention all applications of the theory of utility, in general and the utility functions, in particular due to their variety. It is worth noting that from the point of view of the paper this approach has been borrowed by other disciplines where management and (or) organizational decisions are made. Especially, it concerns technological systems. Considerations of this paper are focused on selected types of networks, i.e. computer and sensor networks where management problems are solved with the use of utility based functions. The paper reports some examples of results obtained in the Division of Intelligent Decision Support Systems of the Wroclaw University of Technology, Poland. The problems presented refer to some very well-known types of analogous decision making problems in Economics, i.e.: investment planning referred also to as capital budgeting or investment appraisal, where a plan for the purchase of resources are determined to minimize costs for required expected incomes (Kalyebara and Islam, 2013, Laux, 1971); deriving of optimal short-term operational (managerial) decisions, called also production planning or production scheduling, where decisions connected with the operation of a company are made to maximize its income for a given amount of resources (Pochet and Wolsey, 2006, Mula *et al*, 2006); and common on-line management of the market situation where the best strategy for a company is chosen in the presence of market competitors.

Emerging of new concepts in computer networks like the autonomic networking and the self-organizing networks entailed the development of automatic or semi-automatic management of computer networks and their resources, e.g. (Bettstetter and Prehofer, 2005, Mortier and Kiciman, 2006). It is strictly connected with the deriving of new management algorithms and computer protocols. Starting from Kelly's seminal works (Kelly, 1997, Kelly's *et al*, 1998), the majority of relevant publications employ the utility based approach.

In the paper we assume a computer network as a set of network facilities like routers, switches, wireless network access points and servers connected by wire or wireless directed links. Such networks are equipped with necessary resources being technical parameters of the facilities, e.g. computational power, capacities of links, storage, e.g. (Tanenbaum and Wetherall, 2010). Nowadays, computer networks are used not only as a medium for the free exchange and distribution of information among end users but, first of all, as commercial service systems when costs and incomes are crucial factors evaluating their work. In a consequence, users and owners (providers, operators) involved in such systems undergo market rules, and appropriate tools have been successfully applied for such technological applications.

Computer network service providers are mainly companies which solve long-term, short-term and on-line management problems. Clients pay for data transmitted and first of all for the quality of the transmission. Every transmission is described by technological parameters like rate, delay, jitter or loss probability. These parameters depend on the amount of resources allocated by a network operator to a particular transmission. A client has to pay more for better transmission parameters. This relationship can be conveniently represented by the appropriate utility function. As the result, the relevant network provider's procedures are launched, and they are carried out in the automatic or semi-automatic way in contemporary computer networks. Unlike non-technological applications where human factors are directly involved to perform relevant management procedures, rigorous and automatically run procedures (algorithms) are expected and specific for technological applications like computer networks.

Three following problems as examples of the mentioned types of management problems are addressed in the paper. The investment planning decisions have to be made at the strategic (long-term) level. First of all, a provider needs to design and plan the structure of a computer network as well as to buy appropriate network devices, i.e.: routers, switches, wireless network access points and servers. Then, the required resources like computational power, capacities of links, storage are acquired. The prospective incomes are usually evaluated based on the utility functions expressing forecasted users' demands which, in a consequence, allow the maximization of income. The minimization of costs of the network resources is considered while the minimal acceptable level of income is preserved (see Gąsior and Turowska, 2006).

For a given network infrastructure, i.e. for network devices and interconnections among them, the short-term decisions consist in such an allocation of available resources to maximize incomes resulting from the users utilities (see Kelly, 1997 and Gąsior, 2008). In the paper, the fundamental utility based resource allocation problems for computer networks are presented as the example of short-term management problems.

Finally, the computer network service providers have to take into account the competition on a computer market among different operators which try to provide services for consumers. On the other hand, the co-operation among the operators is indispensable to fulfil users' requirements. This situation can be also modelled using the utility function approach like for the corresponding on-line management cases in economic systems.

In fact, two latter problems differ with respect to the time horizon as well as to the applied solution method, as it is presented in the subsequent sections. The final result in the form of allocated resources is the same.

1.1 Preliminaries

Let us consider a computer network expressed by a set of links among network nodes. The general notation used for the modelling of computer networks is given in Table I. The most popular class of utility functions

Table I. Notation for computer networks

$l \in \boldsymbol{L} = \{1, 2, \ldots, L\},\ L:$	link index, number of links, respectively,
$r \in \boldsymbol{R} = \{1, 2, \ldots, R\},\ R:$	request index, number of requests, respectively,
$\mathbf{A} = [a_{rl}]:$	binary routing matrix where a_{rl} = 1 (0) if route of *r*th request traverses *l*th link (otherwise),
$\mathbf{x} = [x_r]:$	vector of transmission rates where x_r is the transmission rate of *r*th request,
$x_r^{\min},\ x_r^{\max}:$	minimal, maximal acceptable rate of *r*th request due to Quality of Service (QoS) constraints, respectively,
$w_r:$	utility parameter of *r*th request ('willingness-to-pay'),
$u_r(x_r):$	utility function of *r*th request,
$C_l:$	capacity of *l*th link,
$e_l:$	unit cost of *l*th link capacity,
$h_r(w_r):$	certainty distribution of the utility function parameter w_r,
$h_l(C_l):$	certainty distribution of the link capacity C_l.

$$u_r(x_r) = \begin{cases} w_r (1-\kappa)^{-1} x_r^{1-\kappa}, & \kappa > 0, \quad \kappa \neq 1, \\ w_r \ln x_r, & \kappa = 1, \end{cases}$$

adequate for modelling elastic flows like file transfers using the File Transfer Protocol (FTP), is called isoelastic functions (Mo and Walrand, 2000). For the enhanced inelastic flows like Internet TV, function $u_r(x_r) = w_r \ln(x_r + 1)$ can be used for $x_r \geq x_r^{\min}$. Both functions conform to the requirements imposed on such functions in Economics, e.g. (Ljungqvist and Sargent, 2000, Mykkanen, 1994).

Three decision making problems are considered in the next sections. The first one consists in planning of rate allocations for the network to minimize the investment costs for acceptable levels of the QoS as well as of the total utility. The next problem deals with the short-term management where decisions on feasible rate allocations maximizing the total utility are made. The last problem considered concerns on-line management when different operators should compete and co-operate in the same computer network to fulfil users' demands.

The selected results are illustrated for Polish Optical Internet, Pionier which is the Polish nationwide academic network connecting 26 cities currently consisting of high throughput optical links, Fig.1 [http://www.pionier.net.pl/, online access 17st December 2014]. In this case study, we consider the Pionier as an example of infrastructure capable to implement content provider network functionality. The network can be considered as a backbone for metropolitan area networks. Thus, each city is considered as a routing node, and it is assumed that a caching server is connected to each such node.

Figure 1. Layout of the Pionier computer network

1.2. Uncertain variables

For all investigated management problems, their non-deterministic counterparts are also considered. Such cases are closer to real-world applications where some parameters may be not precisely known and (or) can rapidly change. For example, it concerns the utility parameters w_r which as the users' features can change and are independent on managing subjects. The link capacities C_l and the QoS parameters $x_r^{\min}, x_r^{\max}$ are also subject to variations during the computer networks activities. It is assumed that these uncertainties are represented by the formalism of uncertain variables which is briefly presented (see Bubnicki, 2004), Gąsior and Józefczyk, 2009) for details). The uncertain variables approach can be treated as the special case of the fuzzy approach for numerical parameters in the case when their exact values are fixed and certain, but they are not known for the user. Two soft properties are used in the definition of the uncertain variable $\bar{w}_r$: '$\bar{w}_r \cong w_r$' which means '$\bar{w}_r$ is approximately equal to w_r' and '$\bar{w}_r \,\tilde{\in}\, D_r$' which means '$\bar{w}_r$ approximately belongs to the set D_r'.

The uncertain variable $\bar{w}_r$ is defined by a set R of its real number values, the function $h_r(w_r) = v[\bar{w}_r \cong w_r]$ referred to as the certainty index given by an expert and the following definitions for any $D_r, D_1, D_2 \subseteq \mathsf{R}$:

$$v(\bar{w}_r \,\tilde{\in}\, D_r) = \begin{cases} \max\limits_{w_r \in D_r} h_r(w_r), & \text{for} \quad D_r \neq \varnothing, \\ 0, & \text{for} \quad D_r = \varnothing, \end{cases}$$

$$v(\bar{w}_r \,\tilde{\in}\, D_r) = 1 - v(\bar{w}_r \,\tilde{\notin}\, D_r),$$

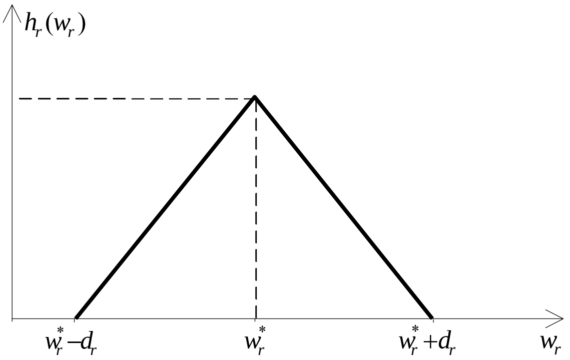

Figure 2. Triangular certainty distribution

$$v(\overline{w}_r \mathrel{\tilde{\in}} D_1 \vee \overline{w}_r \mathrel{\tilde{\in}} D_2) = \max\{v(\overline{w}_r \mathrel{\tilde{\in}} D_1), v(\overline{w}_r \mathrel{\tilde{\in}} D_2)\},$$

$$v(\overline{w}_r \mathrel{\tilde{\in}} D_1 \wedge \overline{w}_r \mathrel{\tilde{\in}} D_2) = \begin{cases} \min\{v(\overline{w}_r \mathrel{\tilde{\in}} D_1), v(\overline{w}_r \mathrel{\tilde{\in}} D_2)\}, & \text{for} \quad D_1 \cap D_2 \neq \varnothing, \\ 0, & \text{for} \quad D_1 \cap D_2 = \varnothing. \end{cases}$$

The function $h_r(w_r)$ is called a certainty distribution. Usually, it is characterized by two parameters w_r^* and d_r which values indicate respectively the most certain value of the unknown parameter according to the expert's opinion and the range of possible values of the unknown parameter. This function is also characterized by the shape which illustrates the degree of certainty for possible values of the unknown parameter. Figure 2 presents used in the paper triangular certainty distribution when the values of uncertain variable $\overline{w}_r$ belong to the interval $[w_r^* - d_r, w_r^* + d_r]$. The ratio $\gamma_r = \frac{d_r}{w_r^*}$ is often used as the parameter characterizing the degree of expert's uncertainty, i.e. the expert's opinions are more convinced for the less values of γ_r. It is commonly the same for all uncertain variables investigated, i.e. $\gamma = \gamma_r$ for all *r*.

The reminder of paper is organized as follows. Sections 2, 3 and 4 discuss respectively three mentioned management problems, i.e.: investment planning, rate allocation and on-line network operating (management). Both deterministic and uncertain cases are introduced, stated and illustrated using the example of Pionier computer network. The next section extends the investigations on Wireless Sensor Networks (WSNs) where similar management problems are present. Final remarks complete the study.

2. Investment planning problem

In the considered problem, we know in advance the set of requests ***R***, the routing matrix **A**, QoS requirements $x_r^{\min}, x_r^{\max}$, willingness-to-pay parameters w_r, the form of utility functions u_r as well as unit costs e_l. We seek for such a vector of rates $\mathbf{x}^* = [x_r^*]$ which ensures the minimum investment cost and guarantees given satisfactory value α of the total utility. The deterministic version of the problem is defined as follows:

$$\text{minimize} \quad Q_1(\mathbf{x}) = \sum_{l=1}^{L} e_l \sum_{r=1}^{R} a_{rl} x_r, \tag{1}$$

subject to

$$Q_2(\mathbf{x}) = \sum_{r=1}^{R} w_r u_r(x_r) \geq \alpha, \tag{2}$$

$$\forall_r \, x_r^{\min} \leq x_r \leq x_r^{\max}. \tag{3}$$

We have the convex optimization problem due to the form of the utility functions u_r, so the Kuhn-Tucker or other known methods can be used to solve it (for example, see Boyd and Vanderberghe, 2004).

The assumption of precise and full knowledge of all problem parameters is extremely unrealistic. Therefore, the uncertain version is also considered. Some results can be found in (Gąsior and Turowska, 2006). Let us now assume that the willingness-to-pay parameters w_r are not known precisely. We premise also the lack of experimental data necessary to estimate the probability distribution as the objective information of w_r treated as the random variable. To describe this parameter uncertainty, the formalism of uncertain variables has been used which can describe possible values of parameters w_r, being the values of uncertain variables $\overline{w}_r$, in the form of certainty distribution $h_r(w_r)$. In a consequence of this uncertainty, the requirement (2) concerning minimal total utility value in the network may not be fulfilled in the crisp way, but it can be only approximately satisfied according to the acceptable value $\bar{v}$ of the certain index v. Then, the uncertain version of the investment planning problem consists in solving the minimization problem (1) subject to

$$v\left[\sum_{r=1}^{R} \overline{w}_r u_r(x_r) \,\tilde{\geq}\, \alpha\right] \geq \bar{v} \tag{4}$$

and (3) for given $\boldsymbol{R}$, $\mathbf{A}$, $x_r^{\min}, x_r^{\max}$, u_r, e_l as well as $h_r(w_r)$, α and $\bar{v}$.
The solution algorithm presented in (Gąsior and Turowska, 2006) consists in a rather simple determinization of the problem leading to the non-linear optimization solvable by the classical methods.

3. Network rate allocation problem

Other management problems, which are important to solve for the computer networks in a short-term horizon when network infrastructure is ready, consist in the resource allocation. Let us consider the rate allocation as an example. Now, it is assumed that the network infrastructure, i.e. topology $\boldsymbol{L}$ and capacities of links C_l are fixed. Moreover, the set of requests $\boldsymbol{R}$ together with the routing matrix $\mathbf{A}$ are known. QoS parameters $x_{r,\min}$, $x_{r,\max}$ along with functions u_r and willingness-to-pay parameters w_r are also given. The management problem deals with the determination of feasible rate allocation $\tilde{\mathbf{x}} = [\tilde{x}_r]$ to maximize the total utility, i.e.:

$$\text{maximize} \quad Q_2(\mathbf{x}) = \sum\nolimits_{r=1}^{R} w_r u_r(x_r) \tag{5}$$

subject to (3) and

$$\forall_l \sum\nolimits_{r=1}^{R} a_{rl}\, x_r \leq C_l\,. \tag{6}$$

This problem was introduced and the first solution was proposed in (Kelly, 1997). Then, the problem has been intensively studied, see the survey (Palomar and Chiang, 2007).
The uncertain version was also investigated using the uncertain variables approach like in Section 2. The uncertainty concerns now both the utility functions parameters w_r and the link capacities C_l. The non-deterministic character of the 'willingness-to-pay' parameter w_r is rather obvious. The changes in time of the latter parameters may be caused by different factors. Firstly, it is hard to precisely determine the values of demands x_r. Moreover, the values of link capacities C_l offered to users do not contain so called 'protocol overheads', i.e. some amount of capacities necessary to continue the transmission. Additionally, errors in network devices can appear which limit the available capacity, and some network technologies, e.g. wireless transmissions are characterized by volatile values of some parameters. The certainty distributions $h_r(w_r)$ and $h_l(C_l)$ given by an expert are known instead of the precise values of w_r and C_l. It is worth noting that now the uncertainty concerns both the criterion and the constraints, which significantly complicates the problem. The certainty indices are used to express both uncertainties. Finally, the uncertain version of the network rate allocation problem can be stated as follows.

$$\text{maximize} \quad v\left[\left(\sum_{r=1}^{R} \overline{w}_r u_r(x_r) \,\tilde{\geq}\, \alpha\right) \wedge \left(\sum_{r=1}^{R} a_{r1} x_r \,\tilde{\leq}\, \overline{C}_1\right) \wedge \ldots \wedge \left(\sum_{r=1}^{R} a_{rL}\, x_r \,\tilde{\leq}\, \overline{C}_L\right)\right] \tag{7}$$

subject to (3) for given $\boldsymbol{R}$, $\boldsymbol{L}$, $\mathbf{A}$, $x_r^{\min}, x_r^{\max}$, u_r as well as $h_r(w_r)$, $h_l(C_l)$ and α.
The solution algorithm is presented in (Gąsior and Orski, 2013). The difficult optimization with multiple max and min operators is transformed to so-called epigraph form (Boyd and Vandenberghe, 2004) and then solved in the numerical way using the method of successive approximation.

4. Network operating problem

We assume in this section that each link $l \in \boldsymbol{L}$ is operated independently and its capacity C_l has to be divided among all requests traversing it. A part of lth link capacity assigned to rth request is denoted by $x_{r,l}$. The transmission rate x_r of the rth request results from the capacities of all corresponding links belonging to the request's path (route), i.e.: $x_r = \min_{l:a_{r,l}=1} x_{r,l}$. Obviously, the service of request implies the income for a network provider, which is proportional to the transmission rate. Additionally, the provider of *l*th link finds its capacity allocation vector $\mathbf{s}_l \triangleq [x_{1,l}, x_{2,l}, \ldots, x_{R,l}]^{\mathrm{T}}$ solving the following local optimization problem which enable us to have the optimal partial rates for link l $\mathbf{s}'_l = [x'_{1,l}, x'_{2,l}, \ldots, x'_{R,l},]^{\mathrm{T}}$:

$$\text{maximize} \quad Q_3^{(l)}(\mathbf{s}_l) = \sum\nolimits_{r=1}^{R} a_{r,l} w_r u_r (\min_{l:a_{r,l}=1} x_{r,l}) \tag{8}$$

subject to

$$\sum\nolimits_{r=1}^{R} a_{rl} x_{r,l} \le C_l \tag{9}$$

and (3) for given $\boldsymbol{R}$, $\boldsymbol{L}$, $\mathbf{A}$, $x_r^{\min}$, $x_r^{\max}$, u_r, C_l, w_r.

The local optimization problem (8) depends on similar problems for other links in the network possibly owned by other providers. Each provider intends to maximize his (her) own utility by making local decisions on partial rates. Such a form of decision making can be well expressed using the approach of a game theory. Then, the decision making (management) problem, the operator and the feasible solution $\mathbf{s}_l$ are called game, player and strategy, respectively. A vector of strategies of all players is called a strategy profile. The solution of this problem together with the explanation of the way to calculate the Pareto-optimal Nash equilibrium can be found in (Gąsior and Drwal, 2013).

5. Case study

All decision making problems and results of their solutions are presented in the paper using a case study of the Pionier network. Let us assume that an IT company plans to launch a new service consisting in the distribution of multimedia content, e.g. the video transmission to Pionier network's users. The company is located in Warsaw and its consumers live in the south-west of Poland. In a consequence, to start such a new service, the provider needs to establish its own network and to ensure indispensable transmission resources mainly expressed in terms of links' capacities. They may be purchased or leased from a core network, in our example from the Pionier network. It is assumed in the case study that the new service is available for users living in: Warszawa, Łódź, Poznań, Gorzów, Zielona Góra, Częstochowa, Katowice, Opole, Bielsko-Biała, and the network useful for this service has a layout depicted in Fig. 1 with solid lines. The following $R = 8$ transmission requests can appear in the network: Warszawa – Łódź ($r = 1$), Warszawa – Poznań ($r = 2$), Warszawa – Częstochowa ($r = 3$), Warszawa – Katowice ($r = 4$), Warszawa – Opole ($r = 5$), Warszawa – Bielsko-Biała ($r = 6$), Warszawa – Zielona-Góra ($r = 7$), Warszawa – Gorzów ($r = 8$). The requests are transmitted by $L = 8$ links: Warszawa – Łódź ($l = 1$), Łódź – Częstochowa ($l = 2$), Częstochowa – Katowice ($l = 3$), Katowice – Opole ($l = 4$), Katowice – Bielsko Biała ($l = 5$), Łódź – Poznań ($l = 6$), Poznań – Zielona-Góra ($l = 7$), Poznań – Gorzów ($l = 8$). Consequently, the following routing matrix expresses the flow of requests:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}.$$

The first decision-making problem made by the provider for the new service consists in the determination of transmission rates x_r for users and, in a consequence, of the capacities of links C_l minimizing the total investment cost Q_1 and holding the satisfactory level of the total utility, i.e. in the solution of the deterministic version of problem (1). To solve the problem, the willingness-to-pay parameters w_r have to be known. Their

Table II. Deterministic version of problem (1) – the optimal solution

x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	x_6^*	x_7^*	x_8^*	$Q_1(x^*)$	$Q_2(x^*)$
74.51	28.54	12.14	10.62	3.14	4.51	4.15	4.29	243.66	70,00

Table III. Deterministic version of problem (1) – the optimal capacities of links

C_1^*	C_2^*	C_3^*	C_4^*	C_5^*	C_6^*	C_7^*	C_8^*
141.9	30.4	18.27	3.14	4.51	36.98	4.15	4.29

values representing the demand by users for a transmitted content were calculated proportionally to the population of corresponding cities (computer network nodes), namely: $w_1 = 7.18$, $w_2 = 5.5$, $w_3 = 2.34$, $w_4 = 3.07$, $w_5 = 1.21$, $w_6 = 1.74$, $w_7 = 1.2$, $w_8 = 1.24$. The results presented in Tables II and III have been calculated for the following values of other data: the guaranteed value of total utility $\alpha = 70$, the unit lease costs of 1Gbps link capacities $e_l = 1$, $l \in \boldsymbol{L}$, the QoS parameters $x_r^{\min} = 2$, $x_r^{\max} = 100$, $r \in \boldsymbol{R}$, the utility functions $u_r(x_r) = w_r \ln x_r$.

The deterministic problem (1) requires the precise values of parameters w_r which is quite unrealistic due to the fact that the actual values of users' demands are difficult to estimate. Therefore, the uncertain version seems to be closer to the real-world situation. Moreover, such a project as described in the case study is rather unique one. It makes impossible to use very well-known probabilistic approach for the representation of uncertainty. The formalism of uncertain variables introduced in Section 1 which is based on an expert's opinion is fully adequate to the considered example. We assume that the expert expresses his (her) knowledge in the form of triangular distribution (Fig. 2) assuming the values fixed for the deterministic version as $w_r^* = w_r$ and the values of parameters $d_r = 0.1 w_r$. This form of the certainty distribution allows having values of the users' demands from the interval which bounds differ of 10% as the maximum from the most certain values w_r^*. Taking the expert's knowledge into account, we may solve the uncertain version of the investment planning problem (1). The results for different values of the certainty threshold $\bar{v}$ are given in Table IV.

One may notice that the higher risk is taken (the less values of $\bar{v}$) the less cost is incurred. This reasonable relationship is almost linear.

When the network is planned and the service is launched, the sizes of demands w_r can change. Then, the provider should try to maximize the income by changing the transmission rates x_r for the available (leased) links' capacities, i.e. to solve the problem (5). To illustrate such a situation, the simulation has been conducted for the same data like for problem (1) besides the demands w_r which values have been now randomly generated according to the rectangular distribution from the intervals $[w_r - 0.3w_r, w_r + 0.3w_r]$ defined around values w_r for problem (1). The capacities of links C_l are now taken from Table III. The following new values have been generated: $w_1 = 8.12$, $w_2 = 7.15$, $w_3 = 1.65$, $w_4 = 4$, $w_5 = 0{,}85$, $w_6 = 1{,}22$, $w_7 = 0{,}84$, $w_8 = 0{,}85$. The results are given in Table V.

Similarly to the investment planning problem (1), as it has been pointed out in Section 3, it is hard to precisely determine the values of parameters w_r. Moreover, the values of links' capacities C_l can be different than the leased ones. Both uncertainties are taken into account while solving the problem (7), i.e. the uncertain version of (5). They are described by the triangular certainty distributions $h_r(w_r)$ and $h_l(C_l)$ represented respectively by parameters γ_1 and γ_2, defined in Sub-section 1.2. Taking into account such uncertainties, the rate allocation problem (7) was solved for different values of the parameters γ_1 and γ_2 characterizing the expert's knowledge and confidence (the less is value of γ the lower expert's uncertainty). It was assumed that the expert stated as the most certain values of demands w_r^* and of the link capacities C_l^*, i.e. the values being the data and the solutions of the deterministic versions of problem (1), respectively. The results given in the Table VI are limited to the case $\gamma_1 = \gamma_2 \overset{\Delta}{=} \gamma$. It turned out that the difference between ratios γ_1 and γ_2 doesn't change substantially the values of the certainty index v. It is caused by the low level of the required income α ($\alpha = 70$). This small value of the income expected by the provider enables us to have the almost sure optimal result ($v = 0.99$) despite the uncertain values of demands and link capacities. However, the income can change in time, and the solution of problem (7) should be repeated. Table VII presents the results, limited only to the values of certainty index v, for $\alpha = 100$ and different values γ_1 and γ_2. It is easy to see, and it is understandable that the solution (the rate allocation) with the positive values of v, representing the achievement of assumed economic purpose in the form of utility, may not exist. One can notice from Table VII that for low values of γ_1 and γ_2, i.e. for the high risk of the required total utility, $v = 0$ which means that it is not reasonable to expect any revenue. According to the second corollary resulting from Table VII, the credibility of expert's evaluation of w_r is more important than the corresponding evaluation of C_l from the point of view of the certainty of the obtained results, i.e. v.

Table IV. Uncertain version of problem (1) – the optimal solutions for different $\bar{v}$

$\bar{v}$	x_1^*	x_2^*	x_3^*	x_4^*	x_5^*	x_6^*	x_7^*	x_8^*	$Q_1(x^*)$	$Q_2(x^*)$
0.9	72.35	27.71	11.79	10.31	3.05	4.38	4.03	4.16	236.57	70.00
0.7	68.31	26.16	11.13	9.74	2.88	4.14	3.80	3.93	223.39	70.00
0.5	64.65	24.76	10.54	9.21	2.72	3.92	3.60	3.72	211.41	70.00
0.3	61.31	23.48	9.99	8.74	2.58	3.71	3.42	3.53	200.48	70.00
0.1	58.25	22.31	9.49	8.30	2.45	3.53	3.25	3.35	190.49	70.00

Table V. Deterministic version of problem (5) – the optimal solution

$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$	$\tilde{x}_6$	$\tilde{x}_7$	$\tilde{x}_8$	$Q_2(\tilde{x})$
74.51	29.91	12.14	12.03	2.56	3.68	3.51	3.56	77.87

Table VI. Uncertain version of problem (5) – the optimal solutions for $\alpha = 70$ and different $\gamma_1 = \gamma_2 \overset{\Delta}{=} \gamma$

γ	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{x}_3$	$\tilde{x}_4$	$\tilde{x}_5$	$\tilde{x}_6$	$\tilde{x}_7$	$\tilde{x}_8$	$Q_2(\tilde{x})$	v
0.9	74,52	29,91	12,14	12,03	2,56	3,68	3,51	3,56	77,88	0,99
0.7	74,53	29,92	12,15	12,03	2,56	3,68	3,51	3,56	77,90	0,99
0.5	74,55	29,93	12,15	12,04	2,56	3,68	3,51	3,56	77,92	0,99
0.3	74,56	29,93	12,15	12,04	2,56	3,68	3,51	3,56	77,94	0,99
0.1	74,52	29,91	12,14	12,03	2,56	3,68	3,51	3,56	77,88	0,99

Table VII. Uncertain version of problem (5) – the values of v for $\alpha = 100$ and different γ_1, γ_2

$\gamma_1 \backslash \gamma_2$	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.9	0.75	0.74	0.74	0.73	0.73	0,72	0.72	0.7	0.69
0.8	0.74	0.74	0.72	0.71	0.71	0.7	0.68	0.67	0.64
0.7	0.71	0.7	0.69	0.68	0.67	0.66	0.65	0.63	0.6
0.6	0.66	0.67	0.64	0.64	0.62	0.61	0.59	0.57	0.55
0.5	0.64	0,61	0.6	0.58	0.57	0.54	0.52	0.5	0.47
0.4	0.54	0.54	0.5	0.5	0.48	0.45	0.42	0.38	0.34
0.3	0.48	0.44	0.42	0.38	0.35	0.3	0.26	0.2	0.13
0.2	0,28	0.26	0.2	0.16	0.11	0.05	0	0	0
0.1	0	0	0	0	0	0	0	0	0

Table VIII. The optimal solution of problem (8)

x'_1	x'_2	x'_3	x'_4	x'_5	x'_6	x'_7	x'_8	$Q_2(x')$
74.51	29.91	12.14	12.03	2.56	3.68	3.51	3.56	77.87

The rate allocation for computer networks is the real-time process with extremely short processing times. Therefore, it is strongly recommended to carry out it in the decentralized way after the decomposition for individual elements of the network. Now, let us assume that the decisions are made separately for individual links according to the problem (8). The solutions for the considered case study with the same data as before are presented in Table VIII where $\mathbf{x}' = [x'_1, x'_2, \ldots, x'_R]^{\mathrm{T}}$. The capacity allocation vectors s_l can be easily calculated, e.g. $\mathbf{s}_3 = [0,0,0,12.03,2.56,3.68,0,0]$, and the sum of the elements of this vector does not exceed $C_3^* = 18.27$. It is worth noting that obtained results are the same as for the deterministic version of problem (5) (compare the results in Table V). Such results confirm the usefulness of the game theory based solution algorithm presented in (Gąsior and Drwal, 2013), which enable the conservation of the optimality despite the decentralization of the calculations.

6. Other problems

The utility based management problems are also important for wireless sensor networks. A sensor network consists of a set of sensors, i.e. small electronic devices capable to measure data on certain phenomena from a defined area. Sensors transmit later the data to sinks which collect data and provide them to final users. They are usually battery supplied and work in an environment that makes their constant maintenance impossible, so the proper energy management is the crucial task enabling the maximization of the total execution time of WSNs. It does not concern sinks which, unlike sensors, are constantly supplied facilities capable to collect and store measured data. All sensors and sinks, which constitute WSN, are wirelessly connected. The management

Table IX. Notation for WSNs

$m \in \boldsymbol{M} = \{1, 2, \ldots, M\},\ M:$	index of measurement point, number of measurement points, respectively,
$s, r \in \boldsymbol{S} = \{1, 2, \ldots, S\},\ S:$	indices of sensors, number of sensors, respectively,
$j \in \boldsymbol{J} = \{1, 2,\ \ldots, J\},\ J:$	index of sink, number of sinks, respectively,
$x^m_{m,s}:$	size of data collected from measurement point *m* and transmitted to sensor *s*,
$\bar{x}^m_{s,r}:$	size of data collected from measurement point *m* and transmitted from sensor *s* to sensor *r*,
$\tilde{x}^m_{s,j}:$	size of data collected from measurement point *m* and transmitted from sensor *s* to sink *j*,
$e_{m,s}, \bar{e}_{s,r}, \tilde{e}_{s,j}:$	amount of energy consumed by sensor *s* concerning the unit of data for measurement and transmission to sensor, transmissions among sensors and transmissions to sinks, respectively,
$E_s:$	total energy available for sensor *s*,
$w^m:$	utility parameter characterizing data collected at measurement point *m.*

problems presented previously for computer networks can be easily extended for WSNs, see, for example, (Zheng *et al*, 2013) where the profit expressed in the form of utility is connected with a part of link capacity (rate) assigned to different transmissions. However, another approach seems to be more reasonable for the latter networks for which the utility is not connected with the rate of a link capacity but with the importance of data measured and transmitted to the sinks via sensors. Considerations on the corresponding investment planning problem can be found in (Jagusiak and Józefczyk, 2013). Now, the formulation of the network operating problem is presented. Let us focus only on the deterministic version. Unlike previous investigations for the computer networks where the rates were results of decisions, we assume now that routing is not given and paths for data transmission should be designed. WSN considered is composed of sets of measurement points, sensors and sinks referred to as ***M***, ***S*** and ***J***, respectively. The following three individual activities in the network are distinguished: the measurement and transmission of data to the sensor, the transmission of data between different sensors, and the transmission of data from the sensor to the sink. It is assumed that sizes of measured and transmitted data, being the multiple of unit data, are the decision variables presented in Table IX which form the corresponding decision matrices $\mathbf{X}$, $\bar{\mathbf{X}}$, $\tilde{\mathbf{X}}$. All activities need the energy consumption, which expenditure for every sensor is limited. The total utility is proportional to the size of data collected in sinks, and the data received from distinct measurement points can provide the different extent of utility. The network operating problem can be stated as follows:

$$\text{maximize}\ \ Q_2(\mathbf{X}, \bar{\mathbf{X}}, \tilde{\mathbf{X}}) = \sum\nolimits_{m \in \boldsymbol{M}} w^m \Big(\sum\nolimits_{j \in \boldsymbol{J}} u_j(\tilde{x}^m_{s,j}) \Big) \tag{10}$$

subject to

$$\sum\nolimits_{m \in \boldsymbol{M}} \sum\nolimits_{s \in \boldsymbol{S}} x^m_{s,r} = \sum\nolimits_{s \in \boldsymbol{S}} \sum\nolimits_{j \in \boldsymbol{J}} \tilde{x}^m_{s,j}\ , \tag{11}$$

$$\forall_{r \in \boldsymbol{S}} \sum\nolimits_{m \in \boldsymbol{M}} \Big(x^m_{m,r} + \sum\nolimits_{s \in \boldsymbol{S}} \bar{x}^m_{s,r} \Big) = \sum\nolimits_{m \in \boldsymbol{M}} \Big(\sum\nolimits_{s \in \boldsymbol{S}} \bar{x}^m_{r,s} + \sum\nolimits_{j \in \boldsymbol{J}} \tilde{x}^m_{r,j} \Big), \tag{12}$$

$$\forall_{s \in \boldsymbol{S}} \sum\nolimits_{m \in \boldsymbol{M}} \Big(e_{m,s} x^m_{m,r} + \sum\nolimits_{r \in \boldsymbol{S}} \bar{e}_{r,s} \bar{x}^m_{s,r} + \sum\nolimits_{j \in \boldsymbol{J}} \tilde{e}_{s,j} \tilde{x}^m_{s,j} \Big) \le E_s \tag{13}$$

for given $\boldsymbol{M}$, $\boldsymbol{S}$, $\boldsymbol{J}$, $e_{m,s}, \bar{e}_{s,r}, \tilde{e}_{s,j}$, E_s, w^m. Constraints (11), (12) and (13) ensure respectively the data flow conservation and the feasible energy consumption. Any function fulfilling the general requirements mentioned in Section 1 can serve as the utility function $u_j(\tilde{x}^m_{s,j})$, however, the form of

$$u_j(\tilde{x}^m_{s,j}) = \sum\nolimits_{s: \tilde{x}^m_{s,j} \ge 0} \ln(\tilde{x}^m_{s,j} + 1) \tag{14}$$

is recommended.

7. Final remarks

The unified and systematic description of selected results on the utility based management of computer networks has been presented in the paper. Three management problems have been considered corresponding to the analogous problems in Economics. The main difference consisting in the way of performing of management procedures has been pointed out, which are carried out automatically or semi-automatically for computer networks. The concept of utility has been used for the formulation of problems. The case study of Polish Optical Internet, Pionier has been used to illustrate the problems. Some preliminary remarks on the application of presented approach to wireless sensor networks are also given.

Two main directions of further work are the most important. First of all, other management problems for selected classes of computer networks are worth investigating. It concerns the next generation networks like Content Aware Networks and the virtual networks. For example, the data placement problem waits for the solution for the former networks. For the latter networks, the virtual network embedding problem is very important. As it has been mentioned in the paper, the data for the considered problems are not precise and not fully known. Therefore, the solution of non-deterministic versions is fully justified. The application of other representations of the uncertain parameters w_r and C_l, e.g. the pure interval representation without additional description of the uncertainty is planned. All the aforementioned problems are worth considering and some initial results may be found e.g. in (Drwal and Józefczyk, 2014, Gąsior and Drwal, 2011).

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